Closing Wed: HW_9A,9B,9C (9.1/3/4, 3.8)

Final Exam: Saturday, June 4th
1:30-4:20pm
Kane 120

Entry Task:

Record the initial temperature of my hot water at the start of class and 10 minutes into class.

Note:

Concerning differential equations, in this course you are expected to be able to:

1. Solve separable equations.
2. Initial conditions and constants?
3. Understand the applied problems from homework and lecture.

Worried about the applied problems? Go through and look at the applied differential equation on all the old finals (typically the last page).

9.4 Differential Equations Applications

Goal: Look at exponential growth/decay, Newton's law of cooling, and mixing problems.

***1. Law of Natural Growth/Decay*:**

Assumption: "*The rate of growth/decay is proportional to the function value*."

Example:

A population has 500 bacteria at $t = 0$.
After 3 hours there are 8000 bacteria.
Assume the population grows at a rate proportional to its size.
Find $B(t)$.

Example:

The *half-life* of cesium-137 is 30 years.
Suppose we start with a 100-mg sample.
The mass function $m(t)$ decays at a rate proportional to its size.
Find $m(t)$.

2. Newton's Law of Cooling:

Assumption: "*The rate of temperature change is proportional to the difference between the temperature of the object and its surroundings*."

3. Air Resistance:

A skydiver with mass of 60 kg steps out of an airplane.

The initial height is 4,000 meters.

The initial velocity is 0 m/s.

Let y(t) = "height at time *t*"

Let v(t) = y'(t) = "velocity at time *t*"

Note: v'(t) = y''(t) = "accel. At time *t*"

Newton's 2nd Law says:

(mass)(acceleration) = Force

$$m\frac{d^2y}{dt^2} = \text{sum of forces on the object}$$

The force due to gravity has constant magnitude (and it is acting downward):

$F_g = -mg = -60 \cdot 9.8 = -588$ Newtons

One model for air resistance

The force due to air resistance (*drag force*) is proportional to velocity and in the opposite direction of velocity. So

$$F_d = -k\,v \quad \text{Newtons}$$

Assume for this problem k = 12.

4. Mixing Problems:

Assume you have a vat of liquid that has a substance (a contaminant) entering at some rate and exiting as some rate, then

“*The rate of change of the contaminant is equal to the rate at which the contaminant is coming IN minus the rate at which it is going OUT.*”